**Indian Statistical Institute, Bangalore**
B. Math.
First Year, First Semester
Final Examination Date : 5/12/08

Maximum marks: 100 Time: 3 hours

1. Consider the sequences $\{a_n\}_{n\geq 1}, \{b_n\}_{n\geq 1}, \{c_n\}_{n\geq 1}$ where

$$a_n = 1 + (-\frac{1}{5})^n; \quad b_n = (-1)^n + \frac{2}{n}; \quad c_n = \frac{6n+4}{7n-5}.$$

Compute limit superior and limit inferior (as $n$ tends to infinity) for these sequences.
. [15]

2. Let $k : [0,1] \to I\!R$ be a continuous function. Suppose that $k$ has local maximum at two distinct points $x_1, x_2$ in $[0,1]$. Show that $k$ has a local minimum at some point $x_3$ in $[0,1]$. [10]

3. Let $\{f_n\}_{n\geq 1}$ be a sequence of real valued continuous functions on [0, 1] converging pointwise to a continuous function $f : [0,1] \to I\!R$. Show that the convergence is uniform, if

$$f_n(x) \geq f_{n+1}(x) \quad \forall x \in [0,1],$$

for all $n \geq 1$. (Hint: Use compactness of [0, 1]). [15]

4. Let $D$ be a non-empty subset of $I\!R$ and let $h : D \to I\!R$ be uniformly continuous. If $D$ is bounded show that $h$ is bounded. Use this result to show that $g : (0,\infty) \to R$ defined by $g(x) = \frac{1}{x}$ is not uniformly continuous. [15]

5. Let $u : [0,1] \to I\!R$ be a continuous function. Define $v : [0,1] \to I\!R$ by

$$v(x) = \sup\{u(y) : 0 \leq y \leq x\}.$$

Show that $v$ is a continuous function. [15]

6. State and prove mean value theorem. [15]

7. Show that every bounded sequence of complex numbers has a convergent subsequence.
. [10]

8. Consider the series $\sum_{n\geq 1} a_n$, where

$$a_n = \begin{cases} \frac{1}{n^2} & \text{if } n \text{ is odd;} \\ \frac{1}{n^3} & \text{if } n \text{ is even.} \end{cases}$$

Show that this series is convergent but the convergence can not be determined by ratio test or root test. [15]